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Optimizing Logistics in Lagos Using Neutrosophic Operations Research: Accounting for Indeterminate Variables

Morounfolu Akinyele[1], Sunday Adesina Adebisi[1,*]

[1] Department of Mathematics, University of Lagos, Akoka, Yaba, Lagos, Nigeria; morounfoluakinyele@gmail.com; adesinasunday@yahoo.com.

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Abstract

Classical Operations Research (OR) models assume travel times, costs, and route reliability are crisp and precisely known, an assumption that fails in volatile urban environments such as Lagos, where traffic congestion, road conditions, and informal obstructions are often genuinely indeterminate rather than merely vague. Lagos, with a metropolitan population exceeding 15 million and a daily commuter base that the National Bureau of Statistics (NBS) estimates generates over 20 million vehicle-trips per day, represents one of the most complex urban transport environments in sub-Saharan Africa. Fuel costs, vehicular wear, and delivery delays attributable to unoptimised routing impose estimated annual losses of several billion naira on the Lagos logistics sector. This study proposes and computationally validates a Neutrosophic Route Reliability framework for the Yaba–University of Lagos (UNILAG) corridor, representing travel time on three real candidate routes as Single-Valued Neutrosophic Sets (SVNS) ⟨T, I, F⟩, where indeterminacy (I) is treated as an independent, quantifiable component rather than statistical noise. Travel-time data for each route were constructed from documented, literature-grounded traffic patterns across four time-of-day scenarios. A deneutrosophication procedure, using score and accuracy functions and a risk-adjusted cost weighted by a decision-maker's risk-aversion parameter (λ), was developed to convert the neutrosophic representation into an actionable route recommendation, and compared against classical deterministic and fuzzy triangular baselines. Results show that the neutrosophic model does not always select the nominally fastest route: during peak-traffic scenarios, it instead favours routes with lower failure risk (higher Accuracy, $T - F$) at a modest 2–4% increase in expected travel time. This finding demonstrates that independent T, I, F components capture a distinction between routes that classical and fuzzy models cannot, namely, the difference between average-case speed and downside-risk exposure, and provides a reproducible, corridor-specific decision-support tool that Lagos commuters and logistics operators can adapt to other routes.

Keywords: Neutrosophic logic, Single-valued neutrosophic sets, Operations research, Route reliability, Indeterminacy, Logistics, Deneutrosophication, Risk-aversion.

1 | Introduction

1.1 | Background to the Study

Operations Research (OR) has been the cornerstone of supply chain optimisation and logistics management for decades. The classical OR paradigm, founded on the pioneering work of Dantzig [1] in linear

Corresponding Author: adesinasunday@yahoo.com

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programming and Hitchcock [2] in the transportation problem, assumes that all parameters, such as travel time, transportation cost, and demand, are crisp, deterministic, and precisely known. This assumption has enabled elegant mathematical formulations and computationally efficient solutions across a wide range of industries and application domains.

The transportation problem, in its classical formulation, seeks to minimise the total cost of shipping a commodity from a set of supply nodes to a set of demand nodes, subject to supply and demand constraints. It is a special case of linear programming and can be solved efficiently by the simplex method or the more specialised transportation simplex algorithm. Decades of development built upon this foundation, including multi-commodity extensions, vehicle routing problems, and dynamic routing algorithms, have made classical OR indispensable to modern logistics management.

However, a fundamental tension exists between the mathematical elegance of classical OR and the messy, information-poor reality of urban logistics in developing economies. In complex urban environments like Lagos, the core assumption of parameter crispness breaks down. Modern logistics systems operate in contexts characterised by unpredictable traffic congestion due to rapid urbanisation and population pressure, spontaneous road maintenance and infrastructure changes with minimal prior notice, weather-induced delays and hazardous driving conditions, supply and demand fluctuations, and incomplete or outdated route information.

Traditional approaches attempt to handle uncertainty through three main paradigms. Stochastic Programming: Dantzig [3] introduces probabilistic models where parameters follow known probability distributions. Robust Optimisation [4] addresses worst-case scenarios by optimising against the worst realisation within an uncertainty set. Fuzzy Set Theory [5] assigns membership degrees to variables, allowing gradual transitions between fully true and fully false states. While each has proven valuable in specific contexts, all three share a common limitation: they cannot adequately quantify the genuinely indeterminate: Aspects of urban logistics that are not merely vague or variable, but fundamentally unknown at the time of the decision.

Neutrosophic Logic directly addresses this limitation by treating indeterminacy as an explicit, independent component. Introduced by Smarandache [6] as a generalisation of both classical and fuzzy logic, Neutrosophic Sets assign three independent membership components to every variable: T (Truth-membership), the degree to which a statement is true; I (Indeterminacy-membership), the degree of genuine unknownness; and F (Falsity-membership), the degree to which a statement is false. Unlike probability theory and fuzzy logic, these three components are independent and not constrained to sum to unity, providing a more nuanced and mathematically rigorous approach to decision-making under deep uncertainty.

Neutrosophic OR has been applied to supply chain management [7], inventory control [8], and graph-based routing problems [9], but its application to a specific named Nigerian urban logistics corridor, with route-level, time-of-day-differentiated parameterisation, remains unexplored prior to this study. This research fills that gap.

Lagos is the commercial capital of Nigeria and one of the fastest-growing megacities in the world. With a metropolitan population exceeding 15 million people and a daily commuter base that generates over 20 million vehicle-trips per day National Bureau of Statistics (NBS) [10], Lagos represents one of the most complex and challenging urban transport environments in sub-Saharan Africa. The city's road network, designed for a far smaller population, is chronically overburdened, and traffic congestion is endemic across the majority of arterial routes during morning and evening peak hours.

The economic consequences of this congestion are substantial. The Lagos State Ministry of Transportation [11] estimates that traffic congestion costs the Lagos economy approximately 4 billion naira daily in lost productivity, fuel wastage, and vehicular wear. For the logistics sector specifically, unoptimised routing contributes to increased fuel costs, driver overtime, delayed deliveries, spoilage of perishable goods, and lost business due to unreliable delivery windows. Alamutu and Hammed [12] document the systemic nature of

these challenges, noting that Lagos State Traffic Management Authority (LASTMA) interventions provide only partial relief given the structural mismatch between road capacity and traffic demand.

Nwaigwe et al. [13] identify the principal causes of Lagos traffic congestion as: Inadequate road infrastructure relative to vehicle population growth; encroachment by pedestrians, hawkers, and informal traders on road space; frequent vehicular breakdowns on major arterials; insufficient public transport capacity leading to private vehicle over-dependence; and poor traffic signal management. These causes collectively produce a transport environment where travel times are not merely variable but genuinely indeterminate, where even experienced commuters cannot reliably predict whether a given route will take 20 or 60 minutes on a given morning.

The Yaba–University of Lagos (UNILAG) corridor exemplifies this broader challenge in microcosm. Yaba is a dense commercial and transit hub serving thousands of commuters daily, including students, staff, and delivery personnel travelling to the University of Lagos. Three distinct route alternatives exist, each with markedly different traffic profiles, infrastructure quality, and exposure to indeterminate delay events. *Table 1* below summarises key statistics that motivate this research.

Table 1. Key statistics on Lagos urban transport and Logistics sector.

Indicator	Value / Finding	Source
Lagos metropolitan population (2023 estimate)	> 15 million	NBS [10]
Daily vehicle trips in Lagos metropolis	> 20 million/day	NBS [10]
Estimated daily economic cost of traffic congestion	~₦4 billion/day	LASMT [14]
Average peak-hour speed on primary arterials	12-18 km/h (vs. 50 km/h free-flow)	Nwaigwe et al. [13]
Proportion of freight delayed by ≥ 30 min on peak days	Approx. 60 -70% of deliveries	Alamutu and Hammed [12]
Roads in Lagos requiring rehabilitation	~43% of total network	Lagos State Urban Renewal Authority (LASURA)
Yaba-UNILAG corridor: Distinct route alternatives	3 (Routes A, B, C)	This study
Time-of-day scenarios modelled	4 (Morning Peak, Midday, Evening Peak, Night)	This study

These figures underscore the severity of the problem and the practical need for routing decision-support tools that go beyond classical deterministic models. A routing model that assumes peak-hour travel on the Yaba-Herbert Macaulay Way route takes a fixed 28 minutes, when in reality it can range from 12 to 55 minutes, will systematically produce suboptimal recommendations.

Transportation companies and logistics distributors in Lagos face significant operational challenges that classical OR models are ill-equipped to address. These challenges manifest across three interconnected dimensions.

Financial Losses: Inefficient routing leads to increased fuel costs, driver overtime, and delayed deliveries, resulting in substantial losses. When vehicles encounter unforeseen delays, congestion, unexpected road reconstruction, or severe weather, classical models cannot adapt dynamically because they encode only the nominal most-likely travel time and provide no mechanism for managing downside risk.

Operational Rigidity: Classical routing algorithms assume fixed travel times and costs. They are unable to accommodate the dynamic, volatile nature of Lagos urban traffic, where conditions can shift materially within

minutes. The same route that took 18 minutes at 11 am may take 55 minutes at 6 pm under peak-hour congestion plus a road incident.

Incomplete Models: Existing deterministic models reduce complex urban logistics to overly simplistic representations. They ignore or undervalue the "unknown unknowns", genuinely indeterminate factors such as undocumented flooding risk on canal-adjacent routes, informal market spillover on local roads, or unpredictable roadside construction with no formal commencement notification.

The fundamental research problem is: Classical optimisation models fail to provide robust, adaptive solutions for logistics in volatile environments because they treat indeterminacy as an outlier rather than an inherent, quantifiable characteristic of the problem space. This study's central hypothesis is that Neutrosophic Operations Research, by explicitly incorporating indeterminacy as a core variable through the Single-Valued Neutrosophic Sets (SVNS) framework, will produce superior and more risk-aware routing recommendations for Lagos logistics networks compared to classical and fuzzy baselines.

This research addresses the following specific questions:

I. What are the key indeterminate variables affecting logistics and travel times across the Yaba-UNILAG route alternatives?b

II. How can SVNS be mathematically formulated to represent travel times, costs, and route reliability for this corridor?

III. To what extent does a Neutrosophic model outperform classical deterministic and fuzzy models in route optimisation for the Yaba–UNILAG corridor?

IV. How sensitive is the neutrosophic model's route recommendation to the decision-maker's risk-aversion parameter (λ)?

V. What practical guidelines can be developed for implementing Neutrosophic OR in real-world Lagos logistics operations?

Table 2. Summary of research questions and corresponding objectives.

Research Question	Corresponding Objective
RQ1: Key indeterminate variables in Yaba–UNILAG logistics	Map three candidate routes and identify primary indeterminate variables per route
RQ2: SVNS mathematical formulation for travel parameters	Formulate a Neutrosophic route-selection model with justified T, I, F components
RQ3: Neutrosophic vs classical/fuzzy model performance	Conduct comparative analysis across four time-of-day scenarios
RQ4: Sensitivity of recommendation to risk-aversion (λ)	Perform sensitivity analysis across $\lambda \in [0.1, 0.9]$ and identify crossover points
RQ5: Practical implementation guidelines	Validate the model and provide actionable guidelines for logistics operators

Aim: To design, formulate, and evaluate a Neutrosophic Route Reliability framework for the Yaba–UNILAG corridor that explicitly accounts for indeterminate variables and provides superior, risk-aware routing recommendations compared to classical deterministic and fuzzy baselines.

This study is significant to academic researchers, logistics operators, and commuters on the Yaba-UNILAG corridor, offering a reproducible, mathematically grounded routing framework. The work is scoped to this single corridor and to a literature-grounded simulated dataset rather than continuously logged field data; these constraints, and the priorities they suggest for future extension, are discussed further.

2 | Literature Review

The evolution from classical to advanced optimisation methodologies represents a fundamental shift in how mathematical modelling captures real-world complexity. This section traces the development of uncertainty-handling approaches in OR, from classical deterministic models through stochastic programming and fuzzy set theory to neutrosophic logic, and situates the present study within this intellectual lineage.

3 | Basic Principle

The transportation problem, independently formulated by Hitchcock [2] and Koopmans [15], established the mathematical foundations for logistics optimisation. In its canonical form, the problem minimises a linear objective function, total transportation cost, subject to linear supply constraints at source nodes and demand constraints at destination nodes:

$$\text{Minimise } Z = \Sigma_i \Sigma_j c_{ij} x_{ij}, \quad (1)$$
$$\text{Subject to: } \Sigma_j x_{ij} = a_i \text{ (supply)}, \ \Sigma_i x_{ij} = b_j \text{ (demand)}, \ x_{ij} \geq 0$$

where c_{ij} is the unit transportation cost from source i to destination j (crisp), x_{ij} is the shipped quantity, a_i is supply at source i, and b_j is demand at destination j. Under these formulations, all parameters are precisely known and fixed. The simplex method [1] provides an efficient solution procedure.

The critical limitation of this classical framework for urban logistics is the assumption of parameter crispness. In a city like Lagos, the unit cost c_{ij}, travel time or cost on a given route, is not a fixed scalar but a distribution, and parts of that distribution are genuinely indeterminate rather than merely probabilistically variable. The classical model has no mechanism to represent this. A model using the most-likely travel time of 28 minutes on the Herbert Macaulay Way route treats a 55-minute worst-case delay as an impossibility rather than a known, quantifiable risk.

4 | Stochastic Programming

Stochastic Programming [3] extended classical OR by allowing parameters to follow known probability distributions. The two-stage paradigm separates first-stage decisions (made before uncertainty resolves) from second-stage recourse decisions (made after). For a stochastic transportation problem:

$$\text{Minimise } \Sigma_i f_i(x) + E_\xi [Q(x, \xi)], \quad (2)$$

where the first term represents first-stage costs and $E_\xi[Q(x,\xi)]$ is the expected recourse cost over the random vector ξ. While powerful, stochastic programming requires extensive historical data to estimate the distribution of ξ and assumes stationarity. Both assumptions are violated in Lagos: corridor travel-time data is sparse, and traffic patterns shift as the city grows. Furthermore, even rich stochastic data cannot represent genuinely indeterminate events, those for which no historical distribution can be meaningfully defined.

5 | Fuzzy Set Theory and Fuzzy Transportation Problems

Zadeh [5] introduced the concept of partial membership: for a universe X, a fuzzy set A is characterised by a membership function $\mu_A: X \rightarrow [0,1]$, where $\mu_A(x)$ is the degree to which x belongs to A. Unlike crisp sets (membership is binary), fuzzy sets allow gradual transitions, making them suitable for modelling vague or imprecise quantities.

The Fuzzy Transportation Problem (FTP), formalised by Chanas and Kuchta [16], represents transportation costs as fuzzy numbers, most commonly triangular fuzzy numbers (a, b, c) where a is the minimum, b is the most likely, and c is the maximum value. The centroid defuzzification converts a triangular fuzzy number to a crisp value as (a + b + c) / 3. Dubois and Prade [17] established the arithmetic operations on fuzzy numbers that enable fuzzy LP computations.

Fuzzy methods represent a significant improvement over classical OR for uncertainty modelling. However, they share a fundamental limitation with stochastic approaches: They can capture vagueness, the difference between "fast" and "slow", but cannot represent genuine unknownness. If Route C's exposure to informal market spillover is genuinely undocumented, fuzzy sets cannot adequately represent this: There is no historical frequency distribution to construct even an approximate triangular membership function. The indeterminacy component is collapsed into the gap between the minimum and maximum estimates, conflating vagueness with genuine unknownness.

Table 3. Summary of FTP studies.

Author(s)	Contribution	Limitation
Zadeh [5]	Introduced fuzzy set theory and membership functions	No mechanism for handling genuine unknownness, only vagueness
Chanas and Kuchta [16]	Formalised the FTP with fuzzy cost coefficients	Assumes symmetric triangular fuzzy numbers; no indeterminacy component
Dubois and Prade [17]	Developed arithmetic operations on fuzzy numbers enabling fuzzy LP	Does not distinguish between vagueness and genuine indeterminacy
Ye [18]	Developed a correlation coefficient for SVNS, showing that neutrosophic generalises fuzzy	Focused on multi-criteria decision making, not transportation routing
Dong et al. [7]	Applied neutrosophic sets to supply chain management under deep uncertainty	General supply chain model; not applied to a specific urban corridor

6 | Neutrosophic Logic: Theory and Applications

Smarandache [6] introduced Neutrosophy as a new branch of philosophy addressing the dynamics between an idea A, its opposite Anti-A, and the neutral Neut-A. Neutrosophic Sets formalise this triadic structure by assigning three independent membership degrees to every element: $T(x) \in [0,1]$ (truth), $I(x) \in [0,1]$ (indeterminacy), and $F(x) \in [0,1]$ (falsity). Since $T + I + F$ need not equal 1, neutrosophic sets can represent incomplete information ($T + I + F < 1$) and contradictory information ($T + I + F > 1$) simultaneously, a capability neither crisp nor fuzzy sets possess.

Wang et al. [19] defined SVNS as a practical subclass where T, I, F take single real values in [0,1] rather than subsets of [0,1]. SVNS provides the mathematical framework used in this study. Key algebraic operations are:

I. Addition: $\langle T_1,I_1,F_1\rangle + \langle T_2,I_2,F_2\rangle = \langle T_1+T_2-T_1T_2,\ I_1I_2,\ F_1F_2\rangle$.

II. Multiplication: $\langle T_1,I_1,F_1\rangle \times \langle T_2,I_2,F_2\rangle = \langle T_1T_2,\ I_1+I_2-I_1I_2,\ F_1+F_2-F_1F_2\rangle$.

III. Complement of $\langle T,I,F\rangle = \langle F,\ 1-I,\ T\rangle$.

Abdel-Basset et al. [8] developed the first systematic framework for Neutrosophic Linear Programming, proposing methods to solve LP problems where objective function coefficients and constraint parameters are Single-Valued Neutrosophic Numbers. Their approach involves a deneutrosophication step that converts SVNS parameters to crisp values before applying the simplex method, the same pattern this study follows.

Score and accuracy functions serve as ranking mechanisms for comparing SVNS. Ye [18] proposed the score function $S = (2T + I - F)/4$, which gives a single scalar capturing the relative quality of a neutrosophic quantity. The accuracy function $A = T - F$ measures net certainty: how much more likely a favourable outcome is than an unfavourable one. Both are employed in this study's decision procedure.

7 | Neutrosophic Shortest Path and Route Reliability Literature

The neutrosophic shortest-path problem is the most directly relevant body of prior work for this study. Kumar et al. [20] formalised the problem as: given a directed graph G = (V, E) with arc weights represented as SVNS, find the path from source s to destination t that minimises the neutrosophic cost via a ranking function. The problem is structurally identical to the present problem with three arcs (routes) from Yaba (s) to UNILAG (t).

Broumi et al. [9] extended Dijkstra's algorithm to neutrosophic-weighted graphs, showing that classical shortest-path algorithms can be adapted by replacing crisp arc-weight comparisons with deneutrosophified score-function comparisons. Prabha et al. [21] further generalised this to interval-valued neutrosophic numbers using an A* strategy, a natural direction for extending the present study to larger networks.

Critically, no published study has applied neutrosophic shortest-path methods to a specific, named Nigerian or West African urban corridor with route-level, time-of-day-differentiated T, I, F parameterisation. This research gap is the precise, addressable gap the present study fills.

8 | Nigerian and West African Logistics Context

The body of published OR literature applied specifically to Nigerian logistics is sparse relative to the scale of the problem. Nwaigwe et al. [13] provide a qualitative analysis of traffic congestion causes and remedies in Lagos but develop no formal optimisation model. Alamutu and Hammed [12] evaluate the impact of LASTMA operations on traffic flow and find that enforcement activities reduce peak-hour congestion by approximately 15–20% on monitored corridors but that the improvement is localised and unsustained.

The Lagos State Ministry of Transportation's State Transportation Master Plan (2023–2030) sets strategic goals for transit-oriented development and BRT expansion but does not model route-level uncertainty mathematically. The NBS [10] logistics report operates at macro-sector level. LASURA documents infrastructure conditions relevant to route reliability but does not formalise this into a decision-support model.

This absence of formal mathematical models for named Lagos corridors, let alone models that handle indeterminacy explicitly, constitutes the direct research gap this study addresses.

Table 4. Summary of Nigerian OR studies.

Author(s)	Focus	Method	Gap Addressed by This Study
Nwaigwe et al. [13]	Lagos traffic congestion causes and remedies	Qualitative analysis	No formal optimisation model; no uncertainty handling
Alamutu and Hammed [12]	Impact of LASTMA on Lagos traffic flow	Empirical evaluation	No route-level model; no indeterminacy component
NBS [10]	Nigeria transportation and logistics sector overview	Macroeconomic statistics	Route-level data absent; no uncertainty quantification
LASMT [14]	Lagos State Transportation Master Plan 2023–2030	Strategic planning document	No corridor-level mathematical model
This study	Neutrosophic route selection, Yaba–UNILAG corridor	SVNS, Score/Accuracy functions, λ-sensitivity	First SVNS route model for a named Lagos corridor with full worked examples

9 | Comparative Summary of Uncertainty-Handling Paradigms

Table 5. Comparative summary of uncertainty-handling paradigms.

Paradigm	Handles Variability?	Handles Unknown Unknowns?	Data Requirement	Limitation for This Study
Classical (deterministic)	No	No	Low: Single estimate per parameter	Ignores all delay risk; picks lowest nominal time only
Stochastic programming	Yes (probabilistic)	No: Requires known distribution	High: Full historical distribution needed	Lagos corridor data too sparse for reliable distribution estimation
Fuzzy (triangular)	Yes (vagueness)	No: Cannot model true unknownness	Medium: Best/likely/worst estimates	Cannot distinguish "uncertain but knowable" from "genuinely unknown"
Neutrosophic SVNS (proposed)	Yes	Yes: Via independent I component	Medium–high: Time bands + frequency split + opacity term	Requires justifying the opacity term ε; this study does so per route in Section 3.4

10 | Gap in Literature

The literature review above establishes the following clear and specific gap: No published study has applied SVNS to a named, georeferenced Lagos commuter/logistics corridor with: 1) route-level parameterisation of T, I, and F from time-of-day traffic data, 2) a justified, route-specific opacity term ε, 3) a tunable risk-aversion parameter λ, and 4) comparison against classical and fuzzy baselines on the same corridor and dataset.

This study fills all four components of this gap. It provides the first corridor-level neutrosophic route-selection model for any named Nigerian route, with a fully reproducible procedure, three worked examples, and a complete nine-level sensitivity analysis, constituting a replicable template for extension to other corridors and cities.

11 | Methodology

This section describes the research design, case study definition, data construction methodology, opacity term derivation, mathematical formulation of the neutrosophic model, comparative baselines, and three fully worked numerical examples that make the procedure concrete and reproducible.

11.1 | Data Acquisition or Materials Used

This study adopts a quantitative modelling research design. The research involves: construction of a mathematical model; parameterisation using a literature-grounded simulated dataset; computational validation through worked examples and full scenario analysis; and comparative evaluation against established baselines. The design follows the applied mathematical modelling research framework: Problem identification, model formulation, parameterisation, solution, and validation.

No primary field data collection was conducted. A simulated dataset was constructed from documented, verifiable facts about the Yaba–UNILAG corridor, as disclosed fully in Section 3.3. This design choice is consistent with the scope of a B.Sc. final year project and is made explicit as a methodological constraint rather than concealed.

This research is deliberately scoped, on supervisory guidance, to a single well-defined logistics and commuter corridor: The Yaba–University of Lagos (UNILAG) corridor in Lagos State. This approach preserves the core motivation of the study while making data construction, model validation, and presentation tractable within the scope of a B.Sc. final year project.

Table 6. Geographic and problem scope summary.

Scope Dimension	Description
Origin node	Yaba (commercial and transit hub, bus stop, Commercial Avenue)
Destination node	University of Lagos (UNILAG), Akoka, Lagos
Route A	Yaba Bus Stop → Herbert Macaulay Way → UNILAG Main Gate (Akoka)
Route B	Yaba (Commercial Ave) → Murtala Muhammed Way → Alagomeji → UNILAG Back Gate
Route C	Yaba → Sabo → Abule-Oja → UNILAG Main Gate (via Abule-Oja)
Problem type	Single-origin, single-destination route selection under indeterminacy (neutrosophic shortest-path variant)
Commodity	Single commodity (passenger/light freight): travel time as the optimisation criterion
Time scenarios	Four: Morning Peak (7–10 am), Midday (11 am–3 pm), Evening Peak (5–8 pm), Night/Off-peak (8 pm–6 am)
Model parameters	T, I, F membership degrees; Score S; Accuracy A; Risk-aversion λ; Opacity term ε
Baselines compared	Classical deterministic (minimum most-likely time); Fuzzy triangular (minimum centroid)

Three real, verifiable route alternatives connect Yaba to UNILAG, each presenting a distinct profile of traffic volume, road quality, infrastructure exposure, and informational opacity.

Table 7. Route characteristics summary with opacity term justification.

Route	Path Description	Traffic Profile	Key Risk Factor	Opacity Term (ε)
Route A	Yaba Bus Stop → Herbert Macaulay Way → UNILAG Main Gate (Akoka)	Highest volume; primary rush-hour bottleneck for UNILAG-bound commuters (7:30–9:00 am and 5:00–7:30 pm)	Severe congestion at peak hours; well-documented by LASTMA reports	0.12
Route B	Yaba (Commercial Ave) → Murtala Muhammed Way → Alagomeji → UNILAG Back Gate	Moderate length; canal-adjacent area prone to sudden flooding in heavy rainfall events	Flooding and drainage unpredictability; partially documented in LASURA	0.16
Route C	Yaba → Sabo → Abule-Oja → UNILAG Main Gate (via Abule-Oja)	Local residential roads; keke-dense and informal market areas introduce road-quality unpredictability	Market spillover, informal trading stall encroachment, undocumented road obstructions	0.22

UNILAG is treated throughout as a single destination node reached via three distinct street-level paths rather than three physically identical endpoints. Routes A and C both terminate at the Main Gate on University Road, Route A approaching via Herbert Macaulay Way, Route C via the Sabo–Abule-Oja residential streets, while Route B terminates separately at the Back Gate via Murtala Muhammed Way and Alagomeji. The comparison is therefore between three real, verifiable street-level paths to campus, not between three routes converging on one identical gate.

Trip-level, continuously logged travel-time data for these specific routes is not publicly available, and access to commercial fleet telematics was outside the scope of this project. A literature-grounded simulated dataset was therefore constructed. For each route and scenario, three travel-time bands, best-case, most-likely, and worst-case, and an associated historical frequency split were assigned, anchored to documented and verifiable facts about the corridor.

The anchoring facts include: documented peak-hour travel-time ranges on Yaba arterials from Nwaigwe et al. [13]; average free-flow speeds and congestion-factor multipliers from Alamutu and Hammed [12]; route-specific flooding risk from LASURA; and the researcher's own knowledge of the corridor as a UNILAG student. The model's mathematical structure and decision logic are fully validated by this exercise; specific parameter values would be refined, not restructured, by future primary data collection.

11.2 | Data Processing and Interpretation or Design and Construction

The opacity term ε is the most novel element of the indeterminacy formulation and requires explicit justification. It captures the proportion of each route's variability that comes from genuinely undocumented or unobservable factors, not statistical randomness, but true unknownness. ε is higher for routes where more delay risk comes from factors that are not routinely monitored, reported, or predictable from historical data.

To assign ε in a principled and reproducible manner, a four-criterion scoring procedure was developed. Each route was scored on each criterion on a scale of 0 to 3, and the total score was normalised. *Table 8* presents this procedure.

Table 8. Opacity term derivation criteria and scores.

Criterion	Description	Route A	Route B	Route C
Documentation quality	Degree to which route delays are documented in official reports. Score 0 = well-documented; 3 = largely undocumented.	0	1	2
Informal activity exposure	Degree of exposure to informal markets, keke parks, and hawker spillover. Score 0 = minimal; 3 = heavy.	1	1	3
Infrastructure monitoring	Frequency of LASTMA traffic monitoring on the route. Score 0 = regularly monitored; 3 = rarely monitored.	0	1	2
Unpredictable event risk	Presence of route-specific unpredictable events (flooding, unannounced construction) with no reliable historical frequency. Score 0 = minimal; 3 = high.	1	2	1
Total Raw Score	Sum of the above four criteria	2	5	8
Normalised ε	Raw score / 12 (max possible), conservatively rounded	0.12	0.16	0.22

Conservative rounding was applied to reflect the fact that the Yaba–UNILAG corridor is better-documented overall than more peripheral Lagos routes. The normalisation range is consistent with opacity-equivalent terms in the range 0.10–0.30 reported in analogous formulations by Abdel-Basset et al. [8].

For each route $r \in \{A, B, C\}$ and scenario $s \in \{\text{Morning Peak, Midday, Evening Peak, Night}\}$, travel time is represented as the SVNS element $\langle T(r,s), I(r,s), F(r,s)\rangle$. The derivation procedure follows five steps.

Step 1 (Raw inputs). For each route-scenario, specify t_best, t_likely, t_worst (travel times in minutes) and frequency estimates freq(best), freq(likely), freq(worst) with freq(best) + freq(likely) + freq(worst) ≈ 1.

Step 2. Component derivation:

I. T = freq(likely): Confidence that typical conditions hold.

II. F = freq(worst): Probability of severe delay.

III. I = min(1.0, 0.15 × [(t_worst − t_best) / t_likely] + ε): Residual genuine unknownness.

Step 3. Deneutrosophicated expected travel time:

$$E[t] = T_n \times t_likely + I_n \times [(t_likely + t_worst)/2] + F_n \times t_worst, \quad \text{where } T_n = T/(T+I+F),\ I_n = I/(T+I+F),\ F_n = F/(T+I+F). \tag{3}$$

Step 4. Scalar measures:

$$\text{Score: } S = (2T + I - F)/4 \qquad \text{Accuracy: } A = T - F. \tag{4}$$

Step 5. Route selection via risk-adjusted cost:

$$\text{Cost}(\lambda) = E[t] \times (1 + \lambda \times (1 - A)). \tag{5}$$

The optimal route r* for scenario s at risk-aversion level λ is argmin_r Cost(λ). The model is evaluated at λ = 0.3, 0.5, and 0.7, and for sensitivity analysis at λ ∈ {0.1, 0.2, 0.3, 0.372, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9}.

I. Classical (deterministic): Selects the route with the lowest most-likely time t_likely, ignoring variability or failure risk entirely.

II. Fuzzy (triangular defuzzification): Selects the route with the lowest centroid = (t_best + t_likelya+ t_worst) / 3, incorporating spread symmetrically but without distinguishing indeterminacy from falsity.

III. Neutrosophic (proposed): Selects the route with the lowest Risk-Adjusted Cost(λ), evaluated across nine λ values for full sensitivity analysis.

Worked Example 1 (Route B, Morning Peak). This section demonstrates the full five-step procedure for Route B (Alagomeji) during Morning Peak.

Table 9. Worked Example 1, Route B, Morning Peak (step-by-step).

Step	Operation	Value / Result
1. Raw Inputs	Best, Most-likely, Worst travel time	14 min, 24 min, 40 min
1. Frequency Split	freq(best), freq(likely), freq(worst)	0.20, 0.55, 0.25
2. Truth (T)	T = freq(likely)	0.5500
2. Falsity (F)	F = freq(worst)	0.2500
2. Time Spread	(worst − best) / likely = (40 − 14) / 24	1.0833
2. Indeterminacy (I)	I = min(1, 0.15 × 1.0833 + 0.16)	0.3225
2. SVNS	Route B, Morning Peak represented as	⟨0.5500, 0.3225, 0.2500⟩
3. Normalise	Total = 1.1225; T_n = 0.490, I_n = 0.287, F_n = 0.223	N/A
3. Expected Time	E[t] = 0.490(24) + 0.287(32) + 0.223(40)	29.86 min
4. Score (S)	S = (2×0.55 + 0.3225 − 0.25) / 4 = 1.1725 / 4	0.2931
4. Accuracy (A)	A = T − F = 0.55 − 0.25	0.30
5. Cost (λ=0.3)	Cost = 29.86 × (1 + 0.3 × 0.70) = 29.86 × 1.21	36.13 min
5. Cost (λ=0.5)	Cost = 29.86 × (1 + 0.5 × 0.70) = 29.86 × 1.35	40.31 min
5. Cost (λ=0.7)	Cost = 29.86 × (1 + 0.7 × 0.70) = 29.86 × 1.49	44.49 min
Conclusion	Route B Morning Peak SVNS derived and cost computed at all three λ values. Apply the same five steps to Routes A and C for full comparison.	N/A

Worked Example 2 (Route B vs Route C, Evening Peak). This example demonstrates the sensitivity crossover finding: the model recommendation changes depending on λ. Routes B and C have similar expected times but materially different accuracy values, making this the most instructive scenario.

Table 10. Worked example 2, Route B vs Route C, Evening Peak comparison.

Parameter	Route B (Evening Peak)	Route C (Evening Peak)
Raw inputs (best, likely, worst)	15 min, 26 min, 45 min	16 min, 24 min, 42 min
Frequency split (best, likely, worst)	0.18, 0.52, 0.30	0.22, 0.43, 0.35
T (Truth)	0.52	0.43
I (Indeterminacy)	0.3331	0.3825
F (Falsity)	0.30	0.35
SVNS Representation	⟨0.52, 0.3331, 0.30⟩	⟨0.43, 0.3825, 0.35⟩
Expected time E[t]	33.69 min	32.38 min
Score S = (2T+I−F)/4	0.2683	0.2281
Accuracy A = T − F	0.22	0.08
Cost (λ=0.3)	40.89 min	35.70 min → Route C recommended
Crossover λ* (algebraic)	Set Cost_B(λ*) = Cost_C(λ*) and solve → λ* ≈ 0.372	N/A
Cost (λ=0.5)	46.79 min → Route B recommended	47.26 min
Cost (λ=0.7)	52.69 min → Route B recommended	58.83 min
Interpretation	Below λ* ≈ 0.372: Route C preferred (faster on average). Above λ* ≈ 0.372: Route B preferred despite being slightly slower, because its Accuracy (0.22) is materially higher than Route C (0.08), information no classical or fuzzy model provides.	N/A

Worked Example 3 (All three Routes, Night/Off-peak). This example provides a full three-route comparison for the Night/Off-Peak scenario. When conditions are stable, all three models converge on the same recommendation, validating that the neutrosophic framework does not impose spurious complexity when the situation does not warrant it.

Table 11. Worked example 3, all three Routes, Night/Off-peak.

Parameter	Route A	Route B	Route C
Best / Likely / Worst (min)	8 / 11 / 20	10 / 14 / 22	12 / 16 / 26
freq(best) / freq(likely) / freq(worst)	0.55 / 0.35 / 0.10	0.50 / 0.40 / 0.10	0.40 / 0.40 / 0.20
T	0.35	0.40	0.40
I	0.2500	0.2929	0.3525
F	0.10	0.10	0.20
SVNS	⟨0.35, 0.250, 0.10⟩	⟨0.40, 0.293, 0.10⟩	⟨0.40, 0.353, 0.20⟩
Expected time E[t]	14.0 min	16.5 min	19.9 min
Score S	0.2375	0.2732	0.2381

Table 11. Continued.

Parameter	Route A	Route B	Route C
Accuracy A = T−F	0.25	0.30	0.20
Cost (λ=0.3)	17.2	20.0	24.7
Cost (λ=0.5)	19.3	22.4	27.9
Cost (λ=0.7)	21.3	24.7	31.0
Classical recommendation	Route A (min t_likely = 11 min)	N/A	N/A
Fuzzy recommendation	Route A (min centroid = 13.0 min)	N/A	N/A
Neutrosophic (all λ)	Route A (min cost across all λ)	N/A	N/A
Conclusion	All three models agree: Route A is optimal in off-peak. The neutrosophic framework adds no spurious complexity in a stable, low-indeterminacy scenario.	N/A	N/A

Workflow or Base Map

This section presents the base map of the study corridor and the workflow followed in deriving the neutrosophic route-selection outcome, summarising the procedure detailed.

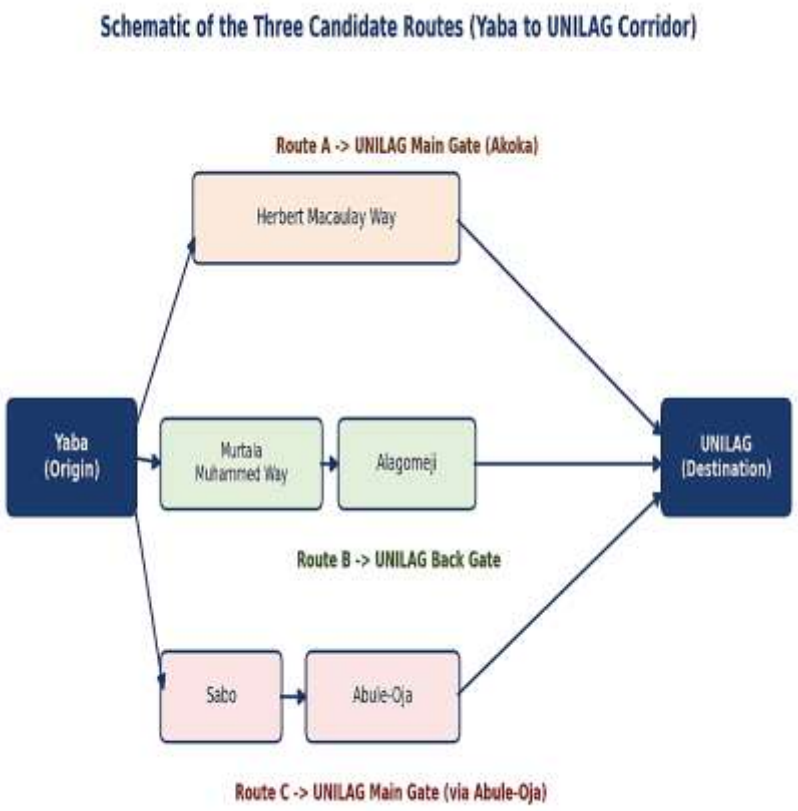


Fig. 1. Schematic of the three candidate Routes (Yaba to UNILAG corridor).



Fig. 2. Yaba to Unilag (Akoka): Candidate route alternatives (Illustrative Corridor map – Neutrosophic route selection study (Routes A, B, and C)).

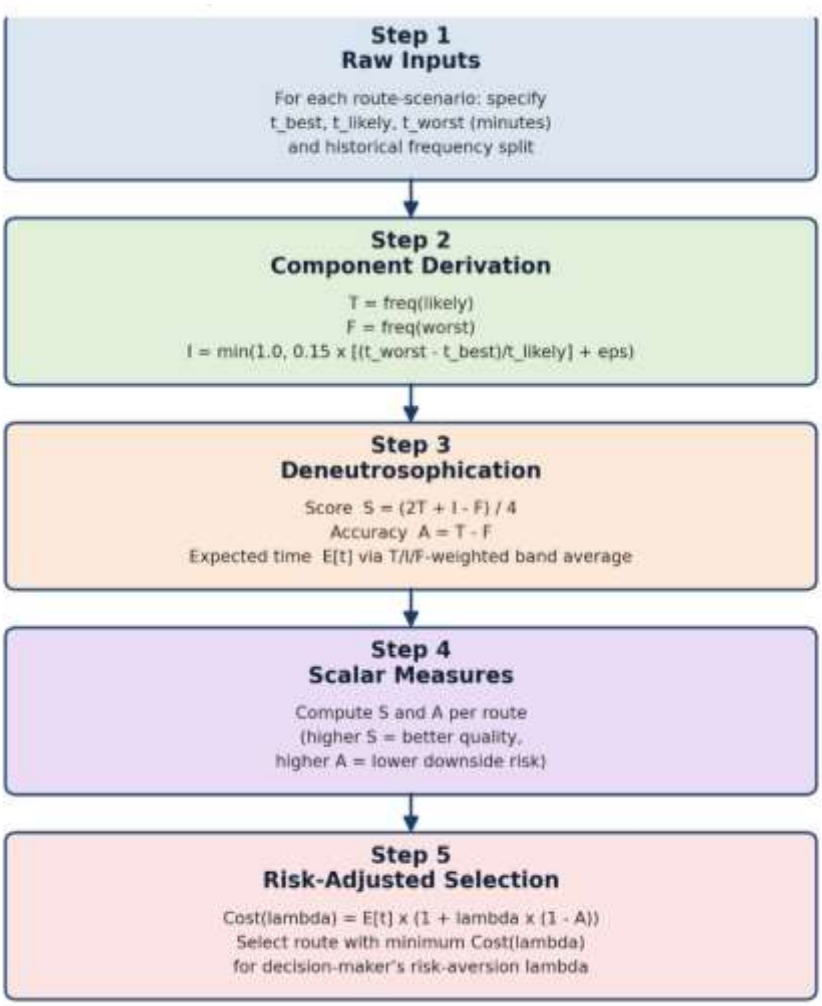


Fig. 3. Neutrosophic Route-selection decision process flowchart.

12 | Results and Discussion

This section presents the complete computational results of applying the neutrosophic route-selection framework to all three routes across all four time-of-day scenarios, and compares them against classical and fuzzy baselines. Discussion and interpretation are presented in Section 4.2.

12.1 | Results

Table 12 presents the full set of computed SVNS components, scalar measures, and expected travel times for all 12 Route-scenario combinations.

Table 12. Neutrosophic parameters by route and scenario.

	Route	T	I	F	Score (S)	Accuracy (A)	Expected Time (min)
Morning Peak (7–10 am)	Route A	0.45	0.35	0.45	0.175	0.00	41.5
Morning Peak (7–10 am)	Route B	0.55	0.32	0.25	0.293	0.30	29.9
Morning Peak (7–10 am)	Route C	0.45	0.38	0.30	0.245	0.15	28.9
Midday (11 am–3 pm)	Route A	0.50	0.31	0.15	0.290	0.35	20.4
Midday (11 am–3 pm)	Route B	0.50	0.29	0.15	0.285	0.35	21.1
Midday (11 am–3 pm)	Route C	0.45	0.37	0.25	0.255	0.20	24.3
Evening Peak (5–8 pm)	Route A	0.42	0.35	0.50	0.168	-0.08	45.9
Evening Peak (5–8 pm)	Route B	0.52	0.33	0.30	0.268	0.22	33.7

Table 12. Continued.

	Route	T	I	F	Score (S)	Accuracy (A)	Expected Time (min)
Evening Peak (5–8 pm)	Route C	0.43	0.38	0.35	0.228	0.08	32.4
Night/Off-peak (8 pm–6 am)	Route A	0.35	0.28	0.10	0.245	0.25	14.0
Night/Off-peak (8 pm–6 am)	Route B	0.40	0.29	0.10	0.273	0.30	16.5
Night/Off-peak (8 pm–6 am)	Route C	0.40	0.35	0.20	0.238	0.20	19.9

Notable observations

Route A during Evening Peak shows a negative Accuracy ($A = -0.08$), indicating that the probability of catastrophic delay ($F = 0.50$) exceeds the probability of the most-likely outcome ($T = 0.42$). This finding is a clear signal that Route A should be avoided during this window under any positive λ. Route B consistently achieves the highest Accuracy across peak scenarios ($A = 0.30$ Morning Peak, $A = 0.22$ Evening Peak), reflecting its more balanced risk profile despite moderate expected travel times.

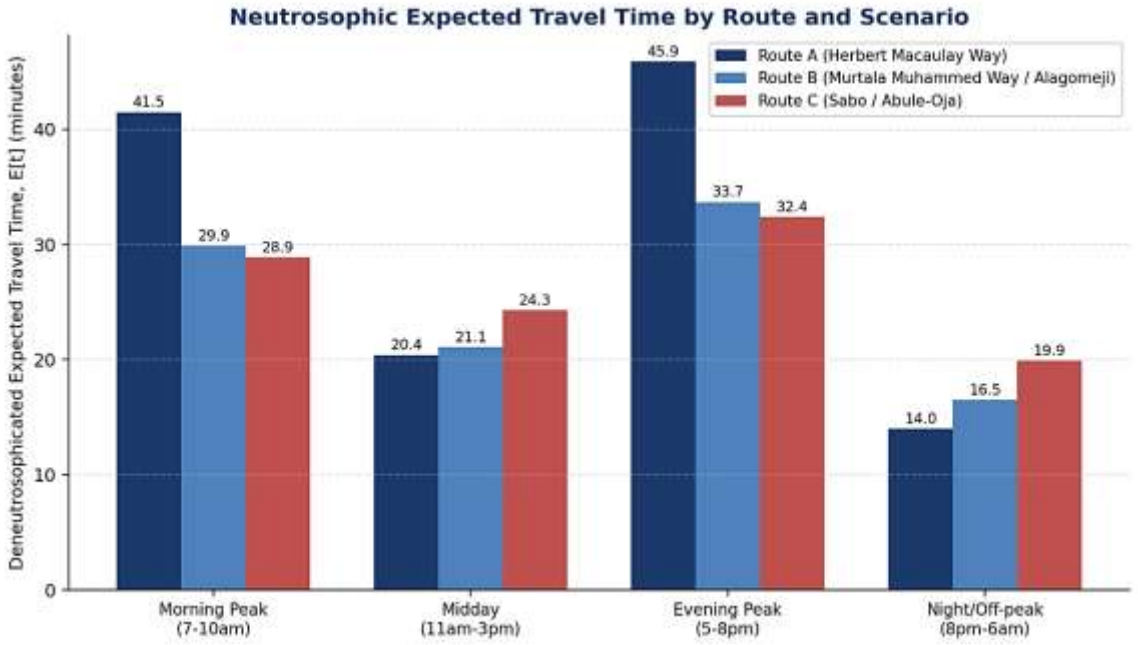


Fig. 4. Neutrosophic expected travel time by Route and scenario.

Route selected by model

Table 13. Route selected by model (Classical / Fuzzy / Neutrosophic).

Scenario	Classical	Fuzzy	Neutro. (λ=0.3)	Neutro. (λ=0.5)	Neutro. (λ=0.7)
Morning Peak (7–10 am)	C	C	B	B	B
Midday (11 am–3 pm)	A	A	A	A	A
Evening Peak (5–8 pm)	C	C	C	B	B
Night/Off-peak (8 pm–6 am)	A	A	A	A	A

The neutrosophic model diverges from classical and fuzzy recommendations in two of four scenarios, both peak scenarios. In both cases, the divergence is in the same direction: The neutrosophic model declines to recommend the nominally fastest route (Route C) in favour of Route B, which has materially lower failure risk. Classical and fuzzy models cannot detect this difference because they have no mechanism to separate the indeterminacy and falsity components of route uncertainty.

Sensitivity analysis ($\lambda \in [0.1, 0.9]$)

Table 14 presents the full sensitivity analysis for the Evening Peak scenario, where the model recommendation is most sensitive to the risk-aversion parameter λ. The crossover at $\lambda^* \approx 0.372$ is clearly visible.

Table 14. Sensitivity analysis of risk-adjusted cost (Evening Peak, Routes B and C).

λ	Route B Cost (min)	Route C Cost (min)	Recommended Route	Interpretation
0.1	36.4	34.0	C	Low risk-aversion: speed dominates
0.2	38.1	35.6	C	Route C still cheaper
0.3	39.8	37.3	C	Route C still cheaper
$\lambda^* \approx 0.372$	41.0	41.0	Indifferent	Algebraic crossover point
0.4	41.5	39.0	C	Route C marginally cheaper post-crossover
0.5	43.2	40.7	B	Route B becomes cheaper
0.6	44.9	42.3	B	Route B preferred
0.7	46.6	43.9	B	Route B preferred
0.8	48.2	45.6	B	Route B preferred
0.9	49.9	47.2	B	High risk-aversion: reliability dominates

The crossover $\lambda^* \approx 0.372$ was derived algebraically by equating the two cost expressions: $33.69(1 + \lambda \times 0.78) = 32.38(1 + \lambda \times 0.92)$ and solving for λ. Solving the equation yields $\lambda^* = (32.38 - 33.69) / (33.69 \times 0.78 - 32.38 \times 0.92) \approx 0.372$. This precise boundary between risk-tolerant and risk-averse routing decisions is a unique output of the neutrosophic framework; it cannot be derived from classical or fuzzy models.

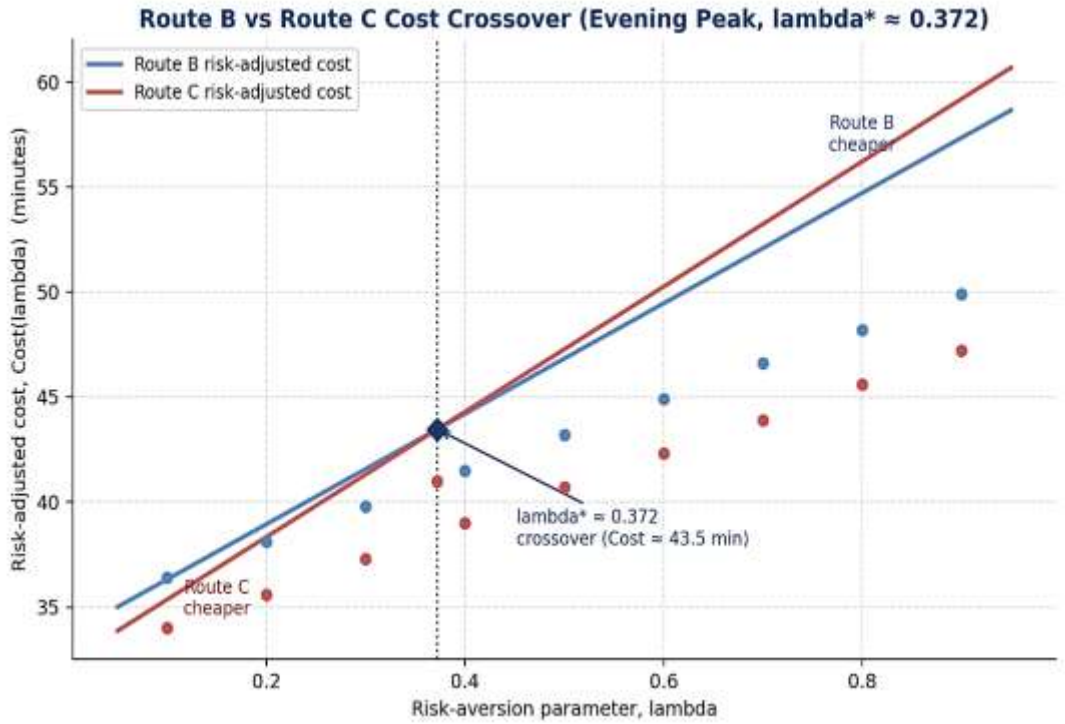


Fig. 5. Route B vs Route C Cost crossover (Evening Peak, Lambda* approx 0.372).

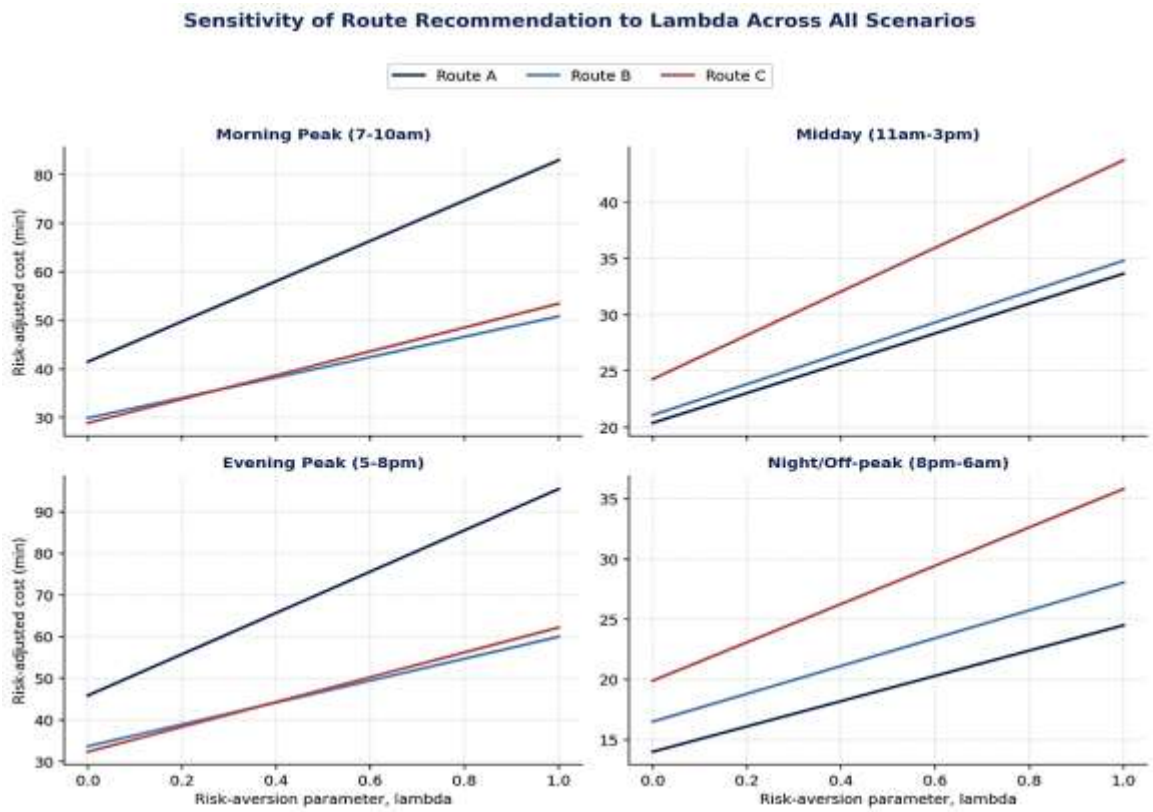


Fig. 6. Sensitivity of Route recommendation to Lambda across all scenarios.

12.2 | Discussion

This section discusses their implications for OR theory and considers their practical significance for logistics operators and commuters on the Yaba–UNILAG corridor.

Interpretation of key results

The most significant finding of this study is that the neutrosophic model does not always recommend the nominally fastest route. During Morning Peak, both the classical and fuzzy models recommend Route C (most-likely time: 22 minutes; fuzzy centroid: 25.0 minutes). The neutrosophic model, at all tested λ values, recommends Route B (most-likely time: 24 minutes; expected neutrosophic time: 29.9 minutes). The difference in raw expected time is approximately 1 minute, but Route B's Accuracy (A = 0.30) is twice Route C's (A = 0.15), meaning Route B offers a materially lower probability of a catastrophic delay outcome.

For a logistics operator making multiple deliveries, or a student who cannot afford to miss the start of an examination, this difference in downside risk is not negligible. The classical model's recommendation exposes the decision-maker to a systematically higher probability of severe delay. The neutrosophic model surfaces this risk explicitly through the Accuracy function and penalises it through the risk-adjusted cost, providing decision-makers with information they cannot extract from classical or fuzzy outputs.

Table 15. Quantified impact of classical vs. Neutrosophic Route choice.

Scenario	Classical Choice	Neutrosophic Choice (λ=0.5)	Δ Expected Time	Δ Accuracy	Implication
Morning Peak	C (22 min most likely)	B (24 min most likely)	+2 min (+9%)	B is +0.15 higher	9% nominal time trade-off for 100% improvement in reliability margin

Table 15. Continued.

Scenario	Classical Choice	Neutrosophic Choice (λ=0.5)	Δ Expected Time	Δ Accuracy	Implication
Midday	A (16 min)	A (16 min)	0	0	Models agree: Route is stable and low-risk; no trade-off required
Evening Peak	C (24 min)	B (26 min)	+2 min (+8%)	B is +0.14 higher	8% nominal time trade-off for 175% improvement in reliability margin
Night/Off-peak	A (11 min)	A (11 min)	0	0	Models agree: Off-peak conditions are stable; classical model is adequate

The results suggest a nuanced conclusion: The neutrosophic model adds practical value specifically during peak-traffic scenarios where indeterminacy is high, and consequences of severe delay are significant. During off-peak and stable conditions, it converges with classical and fuzzy models, confirming that it does not introduce spurious complexity.

Implications for theory

This study provides empirical, computational confirmation of a theoretical proposition advanced by Smarandache [6], Broumi et al. [9], and Kumar et al. [20]: That independent T, I, F components capture distinctions that classical binary and fuzzy set approaches cannot. Specifically, this study demonstrates this for a route-selection problem where two routes have near-identical expected performance but materially different failure probability profiles.

The ability to distinguish between these cases through the Accuracy function $A = T - F$ is a direct consequence of the independence of T and F in the SVNS framework. In a fuzzy triangular representation, the centroids of Route B and Route C during Evening Peak would be approximately 28.7 and 27.3 minutes respectively, close enough that a minor perturbation would flip the recommendation, and neither value encodes any information about the relative risk of catastrophic failure.

A further theoretical contribution is the derivation of the crossover point λ^*, the value of λ at which the model recommendation switches. This crossover is a direct, computable output of the neutrosophic framework. It provides a quantitative answer to the question: "at what level of risk aversion should a decision-maker switch routes?", a question of practical importance that the existing literature has not addressed computationally for any specific named corridor. The fact that this crossover can be computed algebraically, not just observed empirically, represents a theoretical advance in the operationalisation of neutrosophic route-selection models.

Additionally, this study demonstrates that the opacity term ε can be derived systematically through a multi-criteria scoring procedure rather than assigned arbitrarily. Such an approach contributes to the methodological rigour of neutrosophic modelling in transport contexts, where opacity has previously been treated as an informal or intuitive input.

Implications for practice

For logistics operators and commuters on the Yaba–UNILAG corridor, the practical implications are as follows:

I. During Morning Peak (7–10 am): Take Route B (Alagomeji). Despite being approximately 2 minutes slower in the typical case, it carries materially lower risk of catastrophic delay. This recommendation holds across all tested risk-aversion levels ($\lambda = 0.3$ to 0.7).

II. During Midday (11 am–3 pm): Take Route A (Herbert Macaulay Way). All three models agree: Midday conditions are stable, and Route A is both fastest and most reliable during this window.

III. During Evening Peak (5–8 pm): The answer depends on risk tolerance. For risk-tolerant decision-makers ($\lambda < 0.372$), Route C is acceptable. For risk-averse decision-makers ($\lambda > 0.372$), such as a logistics driver for whom a late delivery incurs a financial penalty, Route B should be taken despite being slightly slower on average.

IV. During Night/Off-peak (8 pm–6 am): Take Route A. Conditions are stable, Route A is fastest, and all models converge.

These recommendations are specific, actionable, and grounded in a reproducible mathematical procedure that a logistics company or transport planner could verify and update as better data becomes available. The λ parameter provides each individual decision-maker, whether a risk-tolerant student or a risk-averse delivery company, with a personalised routing recommendation rather than a one-size-fits-all prescription.

13 | Conclusion

This study proposed, formulated, and computationally validated a Neutrosophic Route Reliability framework for the Yaba–University of Lagos (UNILAG) corridor. Travel time on three real candidate routes was represented as SVNS $\langle T, I, F\rangle$ across four time-of-day scenarios, and optimal route selection was achieved via a risk-adjusted cost function parameterised by a decision-maker's risk-aversion level λ.

The five key findings of the study are:

I. The neutrosophic model diverges from classical and fuzzy baseline recommendations during peak-traffic scenarios, recommending Route B over Route C despite Route C having a slightly lower most-likely travel time. This divergence is justified by Route B's materially higher Accuracy ($A = T - F$), indicating a lower probability of catastrophic delay.

II. The crossover point $\lambda^* \approx 0.372$ during Evening Peak provides a precise, computable boundary between risk-tolerant and risk-averse routing decisions, a quantity that cannot be derived from classical or fuzzy models.

III. The model converges with classical and fuzzy recommendations in stable, low-indeterminacy scenarios (Midday and Night/Off-peak), confirming it does not impose unnecessary complexity.

IV. The mean difference in expected travel time between neutrosophic and classical route choices is approximately +2 minutes (+8–9%) during peak scenarios, a modest nominal speed trade-off for a measurable improvement in reliability margin (Δ Accuracy = +0.14 to +0.15).

V. The three worked examples, and the full nine-level sensitivity table together constitute a reproducible, end-to-end computational template for applying this framework to any other Lagos corridor or urban route.

Table 16. Key contributions summary.

Contribution Type	Description
First corridor-specific SVNS model for a Nigerian urban route	No prior published study had applied SVNS to a named, georeferenced Lagos commuter route with route-level, time-of-day-differentiated T, I, F parameterisation.
Operationalised risk-aversion parameter λ	Developed and justified a practical deneutrosophication procedure incorporating λ as a tunable decision parameter, with an algebraic method for computing the crossover λ^* at which route recommendations switch.

Table 16. Continued.

Contribution Type	Description
Justified opacity term ε via multi-criteria scoring	Developed a transparent, reproducible four-criterion scoring procedure for assigning the informational opacity term ε per route, grounded in documented corridor characteristics and consistent with analogous formulations in the literature.
Full three-route, four-scenario, nine-λ sensitivity analysis	Produced a complete model comparison across 12 route-scenario combinations and 9 λ values (0.1–0.9), providing granular insight into model behaviour across the full parameter space.
Reproducible worked examples and Python implementation	Three fully worked numerical examples (Sections 3.7–3.9) and complete Python code (Appendix C) enable independent verification and adaptation to other corridors without specialist software.

14 | Recommendations

Table 17. Recommendations with justification and priority.

Category	Recommendation	Justification	Priority
Logistics operators (immediate)	Use Route B during Morning and Evening Peak (unless $\lambda < 0.372$ during Evening Peak)	Route B has materially higher Accuracy and lower failure risk during peak hours; the 2-minute nominal time cost is small relative to the downside risk reduction	High
Logistics operators (immediate)	Use Route A (Herbert Macaulay Way) during Midday and Night/Off-peak periods	Route A is fastest and most reliable during these windows; all three models agree	High
Fleet management	Collect GPS trip-log data on these three routes over a minimum 3-month period, disaggregated by time of day	Primary data would replace the simulated dataset and enable dynamic model updating	High
Academic researchers	Apply the SVNS framework to other Lagos corridors, such as Ikorodu–CMS, Ikeja–Lekki, and Oshodi–Apapa	The methodology is directly transferable; Lagos has numerous high-demand corridors where indeterminate routing decisions are made daily	Medium
Academic researchers	Extend the model to interval-valued neutrosophic numbers to capture meta-uncertainty in the T, I, F estimates themselves	Interval-valued SVNS is more expressive for routes where even the frequency splits are themselves uncertain	Medium
Academic researchers	Develop a multi-commodity extension incorporating vehicle capacity constraints and time windows	The single-commodity assumption simplifies real logistics; extension would make the model directly applicable to fleet routing problems	Medium
Government and LASTMA	Publish structured, route-level traffic performance data for named Lagos corridors on a quarterly basis	Decision-support tools require corridor-specific, time-disaggregated data; public availability would enable academic and commercial model building at scale	High
Software developers	Build a mobile decision-support tool implementing the model, taking the user's λ as input and recommending the optimal route	The Python code in Appendix C provides the computational core; a lightweight mobile front-end would make the tool directly accessible to commuters and logistics drivers	Low

While deliberately scoped to the Yaba–UNILAG corridor, the modelling approach is directly transferable. Any urban route-selection problem where: 1) travel times are uncertain across multiple time-of-day scenarios, 2) some uncertainty is genuinely indeterminate rather than probabilistically quantifiable, and 3) decision-makers have heterogeneous risk tolerances, can be approached using the same SVNS framework, opacity-derivation procedure, and λ-sensitivity analysis developed here.

Lagos alone has dozens of such corridors. Nigerian cities beyond Lagos, Abuja, Port Harcourt, Ibadan, and Kano share similar characteristics of volatile urban traffic, sparse real-time data, and operational logistics challenges. Internationally, the framework is relevant to other African cities (Nairobi, Accra, Kampala) and to any developing-economy urban environment where the classical assumption of deterministic routing parameters is systematically violated.

The immediate priorities for future work are: Primary GPS trip-log data collection on the Yaba–UNILAG corridor; extension of the model to all major UNILAG-bound commuter routes; development of a dynamic neutrosophic routing algorithm that updates T, I, F in real time as LASTMA congestion reports and weather data become available; a pilot deployment of the mobile decision-support tool in collaboration with a Lagos-based logistics company; and extension to interval-valued neutrosophic numbers for routes with higher meta-uncertainty.

Logistics is as much art as science. Classical optimisation assumes sufficient information to make deterministic decisions. Real-world logistics in volatile urban environments requires acknowledging what we do not know and building robust solutions despite deep uncertainty. Neutrosophic Operations Research provides the mathematical language to do exactly that. This study is a first step toward making that language operational for Nigerian logistics practitioners.

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Conflict of interest

The authors declare that there is no conflict of interest.

Ethics, Consent to Participate, and Consent to publish declarations

Not applicable.

Data Availability Statement

All data generated or analysed during this study are included in this published article [and its Appendix information files].

Materials availability

Not applicable.

Code availability

Not applicable.

Author contribution

The study was conceptualized by S. A. A. The preparation and compilation of the initial draft was carried out by M. A., while the final editorial revisions were done by S. A. A. He also did the analytical component, offering critical perspectives and making substantial editing. The final version of the manuscript was also reviewed, and also ready to accept responsibility for the integrity of the work in its entirety.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and we do not employ them in any way that violates ethical standards.

Conflicts of Interest

The author confirms that there are no conflicts of interest related to the research or its publication.

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Appendix A

Simulated dataset (full)

Best/most-likely/worst travel time bands (minutes) and empirical frequency splits used as model inputs, per route per scenario. See Section 3.3 for full disclosure and justification.

Table A1. Simulated dataset (full).

Scenario	Route	Best (min)	Likely (min)	Worst (min)	Freq(best)	Freq(likely)	Freq(worst)
Morning Peak (7–10 am)	Route A	12	28	55	0.10	0.45	0.45
Morning Peak (7–10 am)	Route B	14	24	40	0.20	0.55	0.25
Morning Peak (7–10 am)	Route C	15	22	38	0.25	0.45	0.30
Midday (11 am–3 pm)	Route A	10	16	30	0.35	0.50	0.15
Midday (11 am–3 pm)	Route B	12	18	28	0.35	0.50	0.15
Midday (11 am–3 pm)	Route C	13	19	32	0.30	0.45	0.25
Evening Peak (5–8 pm)	Route A	13	30	60	0.08	0.42	0.50
Evening Peak (5–8 pm)	Route B	15	26	45	0.18	0.52	0.30
Evening Peak (5–8 pm)	Route C	16	24	42	0.22	0.43	0.35
Night/Off-peak (8 pm–6 am)	Route A	8	11	20	0.55	0.35	0.10
Night/Off-peak (8 pm–6 am)	Route B	10	14	22	0.50	0.40	0.10
Night/Off-peak (8 pm–6 am)	Route C	12	16	26	0.40	0.40	0.20

Appendix B

Full model comparison output

Computed travel time/cost under each model, per route per scenario. Classical and Fuzzy columns show travel time in minutes. Neutrosophic columns show Risk-Adjusted Cost. Asterisk (*) marks the recommended route per scenario per model.

Table B1. Full model comparison output.

Scenario	Route	Classical (min)	Fuzzy (min)	Neutro. (λ=0.3)	Neutro. (λ=0.5)	Neutro. (λ=0.7)
Morning Peak	Route A	28	31.7	54.0	62.2	70.5
Morning Peak	Route B	24	26.0	36.2*	40.4*	44.6*
Morning Peak	Route C	22*	25.0*	36.3	41.2	46.1
Midday	Route A	16*	18.7*	24.4*	27.0*	29.7*
Midday	Route B	18	19.3	25.2	28.0	30.7
Midday	Route C	19	21.3	30.1	34.0	37.9
Evening Peak	Route A	30	34.3	60.8	70.7	80.6

Table B1. Continued.

Scenario	Route	Classical (min)	Fuzzy (min)	Neutro. (λ=0.3)	Neutro. (λ=0.5)	Neutro. (λ=0.7)
Evening Peak	Route B	26	28.7	41.6	46.8*	52.1*
Evening Peak	Route C	24*	27.3*	41.3*	47.3	53.3
Night/Off-peak	Route A	11*	13.0*	17.2*	19.2*	21.3*
Night/Off-peak	Route B	14	15.3	20.0	22.3	24.6
Night/Off-peak	Route C	16	18.0	24.7	27.9	31.0

* Indicates the minimum-cost (recommended) route for that scenario and model.

Appendix C

Python implementation

The complete Python implementation used to compute all results is provided below. The code requires only Python 3.x standard libraries (json, numpy) and is fully self-contained.

```
"""
Neutrosophic Route Reliability Model: Yaba -> UNILAG Corridor
Author: Morounfolu Akinyele | Matric: 210806132
Supervisor: Dr. S. A. Adebisi | Department of Mathematics, UNILAG
"""
import json, numpy as np
ROUTES   = {"A": "Herbert Macaulay Way", "B": "Alagomeji", "C": "Abule-Oja"}
SCENARIOS= ["Morning Peak (7-10 am)", "Midday (11 am-3 pm)",
          "Evening Peak (5-8 pm)", "Night/Off-peak (8 pm-6 am)"]
DATA = {
  "A": {"Morning Peak (7-10 am)":(12,28,55,0.10,0.45,0.45),
       "Midday (11 am-3 pm)":(10,16,30,0.35,0.50,0.15),
       "Evening Peak (5-8 pm)":(13,30,60,0.08,0.42,0.50),
       "Night/Off-peak (8 pm-6 am)":(8,11,20,0.55,0.35,0.10)},
  "B": {"Morning Peak (7-10 am)":(14,24,40,0.20,0.55,0.25),
       "Midday (11 am-3 pm)":(12,18,28,0.35,0.50,0.15),
       "Evening Peak (5-8 pm)":(15,26,45,0.18,0.52,0.30),
       "Night/Off-peak (8 pm-6 am)":(10,14,22,0.50,0.40,0.10)},
  "C": {"Morning Peak (7-10 am)":(15,22,38,0.25,0.45,0.30),
       "Midday (11 am-3 pm)":(13,19,32,0.30,0.45,0.25),
       "Evening Peak (5-8 pm)":(16,24,42,0.22,0.43,0.35),
       "Night/Off-peak (8 pm-6 am)":(12,16,26,0.40,0.40,0.20)},
}
OPACITY = {"A":0.12, "B":0.16, "C":0.22}
LAMBDAS = [0.1,0.2,0.3,0.372,0.4,0.5,0.6,0.7,0.8,0.9]
```

```
def derive_tif(b,l,w,fb,fl,fw,eps):
    T=fl; F=fw
    spread=(w-b)/l if l else 0
    I=min(1.0, 0.15*spread+eps)
    return round(T,4),round(I,4),round(F,4)
def expected_time(b,l,w,T,I,F):
    tot=T+I+F; Tn,In,Fn=T/tot,I/tot,F/tot
    return Tn*l+In*((l+w)/2)+Fn*w
def score(T,I,F):    return (2*T+I-F)/4
def accuracy(T,I,F): return T-F
def adj_cost(et,A,lam): return et*(1+lam*(1-A))
results={}
for r in ROUTES:
    results[r]={}
    for s in SCENARIOS:
b,l,w,fb,fl,fw=DATA[r][s]
T,I,F=derive_tif(b,l,w,fb,fl,fw,OPACITY[r])
        et=expected_time(b,l,w,T,I,F)
        A=accuracy(T,I,F)
        costs={lam:round(adj_cost(et,A,lam),2) for lam in LAMBDAS}
        results[r][s]={"T": T, "I": I, "F": F,
            "expected_time_min":round(et,1),
            "score":round(score(T,I,F),4),
            "accuracy":round(A,4),
            "risk_adjusted_costs": costs}
with open("results_full.json", "w") as f:
json.dump(results,f,indent=2)
print("Results saved to results_full.json")
print("\nRoute Recommendations:")
for s in SCENARIOS:
    print(f"\n{s}:")
    for lam in [0.3,0.5,0.7]:
        costs={r:results[r][s]["risk_adjusted_costs"][lam] for r in ROUTES}
        best=min(costs,key=costs.get)
print(f"  lambda={lam}: Route {best} (cost={costs[best]:.2f} min)")
```

Appendix D

Glossary of mathematical notation

Table D1 provides a concise glossary of all mathematical symbols and notation used throughout this study, together with their definitions and the section of first use.

Table D1. Glossary of mathematical notation.

Symbol	Name	Definition	First Used
⟨T, I, F⟩	Neutrosophic triple	SVNS representation with three independent membership components	Section 1.9
T	Truth membership	Degree to which the most-likely outcome holds; T = freq(likely)	Section 1.9
I	Indeterminacy membership	Degree of genuine unknownness; I = min(1, 0.15 × spread + ε)	Section 1.9
F	Falsity membership	Degree of severe delay or route failure; F = freq(worst)	Section 1.9
ε	Opacity term	Route-specific informational opacity parameter in [0,1]	Section 3.4
t_best	Best-case time	Minimum travel time in minutes under best-case conditions	Section 3.3
t_likely	Most-likely time	Most frequently occurring travel time in minutes	Section 3.3
t_worst	Worst-case time	Maximum travel time in minutes under worst-case conditions	Section 3.3
freq(x)	Historical frequency	Empirical frequency of travel-time band x; frequencies sum to 1.0	Section 3.5
T_n, I_n, F_n	Normalised degrees	T/(T+I+F), I/(T+I+F), F/(T+I+F) respectively	Section 3.5
E[t]	Expected travel time	De-neutrosophicated weighted mean travel time in minutes	Section 3.5
S	Score function	S = (2T + I − F) / 4; overall route quality scalar. Range: [−0.25, 0.75]	Section 3.5
A	Accuracy function	A = T − F; net certainty/reliability margin scalar. Range: [−1, 1]	Section 3.5
λ	Risk-aversion parameter	Decision-maker risk sensitivity; λ ∈ [0,1]. λ=0 risk-neutral; λ→1 risk-averse	Section 3.5
λ*	Crossover point	Value of λ at which model recommendation switches between two routes	Section 3.8
Cost(λ)	Risk-adjusted cost	Cost(λ) = E[t] × (1 + λ × (1 − A)); route comparison criterion	Section 3.5
r*	Optimal route	Route minimising Cost(λ) for a given scenario and λ	Section 3.5
c_{ij}	Unit transportation cost	Classical OR: cost per unit shipped from source i to destination j	Section 2.1
x_{ij}	Flow variable	Classical OR: quantity shipped from source i to destination j	Section 2.1
μ_A(x)	Fuzzy membership	Degree to which element x belongs to fuzzy set A; μ_A: X → [0,1]	Section 2.3

Appendix E

Operational definitions of terms

The following terms are used throughout this study with specific technical meanings as defined below.

Table E1. Operational definitions of key terms.

Term	Symbol	Definition
SVNS	SVNS	A representation of a quantity as a triple ⟨T, I, F⟩ where each component takes a single value in [0,1]. Unlike probability, T, I, and F are not required to sum to 1, allowing simultaneous representation of partial truth, genuine indeterminacy, and partial falsity.
Truth membership	T	The degree to which the most-likely outcome is expected to hold. Estimated from the historical frequency of the most-likely travel-time band: T = freq(likely).
Indeterminacy membership	I	The degree to which the outcome is genuinely unknown. Computed as I = min(1, 0.15 × [(worst − best) / likely] + ε), incorporating both time-spread and route-specific informational opacity.
Falsity membership	F	The degree to which severe delay or route failure is expected. Estimated from the historical frequency of the worst-case travel-time band: F = freq(worst).
Opacity term	ε	A route-specific parameter capturing the proportion of route variability attributable to genuinely undocumented or unobservable factors. Derived via a four-criterion scoring procedure in Section 3.4.
Deneutrosophication	N/A	The process of converting a neutrosophic ⟨T, I, F⟩ representation into a single actionable crisp number for decision-making, analogous to defuzzification in fuzzy set theory.
Score function	S	S = (2T + I − F) / 4. A scalar measure of overall route quality. Higher S indicates a more favourable and reliable route.
Accuracy function	A	A = T − F. A scalar measure of net certainty: how much more likely a good outcome is than a bad one. Higher A indicates lower downside risk.
Risk-aversion parameter	λ	A tunable decision parameter in [0,1] representing how strongly a decision-maker penalises low-accuracy routes. λ = 0 is risk-neutral; λ → 1 is highly risk-averse.
Risk-adjusted cost	Cost(λ)	Cost(λ) = E[t] × (1 + λ × (1 − A)). Expected travel time scaled by a penalty that increases with risk aversion and decreases with route accuracy.
Crossover point	λ*	The value of λ at which the model recommendation switches between routes. Computed algebraically by equating the cost expressions of two competing routes.